

§2. Synopsis Propositions 2 - 6: The Law of Refraction for Dense Ellipsoids and Hyperboloids.

According to Gregory, for a ray with a given angle of incidence i at a plane boundary separating two media, the angle of deviation d of the ray as it passes from one medium to the other is a measure of the refraction between the surfaces.

In *Prop. 2*, an experiment is described in which the angles labeled i and d are measured.

Prop. 3 shows how to construct an ellipse in which the incident ray is parallel to the principle axis at some point on the ellipse, with an angle i to the normal of the tangent at that point, while the refracted ray at an angle $r = i - d$ passes through the far focus. Note that new ideas introduced are called *Theorems*, while applications of given material are called *Problems*.

Prop. 4 demonstrates that the ratio $\sin i : \sin r$ reduces to the ratio axis length : inter-focal distance.

Prop. 5 extends the result of *Prop. 4* to the case of many rays parallel to the axis, which all pass through the far focus of the dense ellipsoid. There are subsequently some tables for refraction through water, glass, etc, taken from the works of Witelo, Kercher, which he compares with his own measurements. Gregory appears to have conducted very precise experiments of his own. Of interest to the modern reader is the practice of comparing all the measurements to those of a particular angle, rather than taking an average, for the actual idea of a refractive index was not yet in use.

Prop. 6 introduces the other kind of refracting surface - related to the hyperbola. In the ellipsoidal case, rays parallel to the axis are sent through the far focus by refraction at the elliptic interface. In this case, rays diverging in air from the far focus are rendered parallel on refraction at the hyperbolic interface of the dense medium.

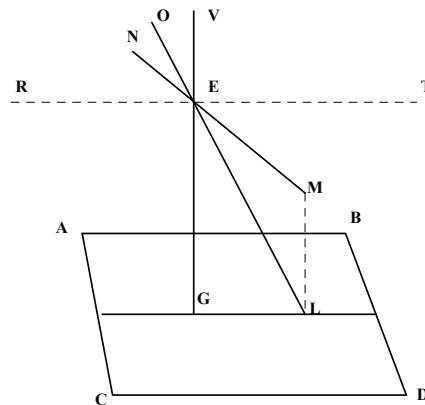
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§2. Prop.2.1

Prop. 2. Problem.

To find the refraction of any medium with air.

Let some plane ABCD be set up parallel to the horizontal or close to it ; and let some other point be fixed at a higher position E, and through E a perpendicular VEG may be considered to be drawn to the horizontal, and the angle of incidence in the medium (of which the refraction of the angle in the medium is required) is GEL, which is measured with an astrolabe or quadrant. Finally the whole space between the plane ABCD and the point E is filled up a medium; the smooth surface of which shall be accurately parallel to the horizontal at the point E.



Prop.2-Figure 1.

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By viewing with the eye placed at E, a small body placed at L shining brightly will appear to be shining brightly at M. Therefore, by measuring the angle NEV, [or GEM], the difference between that angle and the first angle OEV, [or GEL] will give the angle of refraction NEO sought, coming together with the angle of incidence NEV in air at E : [the task] which had to be accomplished. Anyone who wishes to find the angles of refraction by other means, may consult Witelo, Kepler, and the other dioptrics authorities.

§2. Prop.2.2.

Note on Prop. 2.

The Law of Refraction as we know it, had first been established experimentally by Thomas Harriot in the summer of 1601, but he had not communicated his discovery beyond a close circle of friends that included Aylesbury and Warner. [*vide* J. Lohne, *Essays on Thomas Harriot*, J. Arch. Exact Sciences, p.275,(1979)]: the law was to be rediscovered by Snell in 1624, but was not published by him. Descartes (1637) had independently discovered the law experimentally, while Fermat had applied his principle of least time to give the first theoretical explanation of the phenomena of refraction and reflection. Thus the scientific community, such as it was at the time, was familiar with Snell's Law when Gregory produced his book. Gregory however did not have the law of refraction as a ratio of sines, though he measured refraction with a ratio that can be reduced to this form for the case of conoidal surfaces. The method adopted by Gregory to measure the angle of refraction of a ray, say through a flat glass slab, appears to be as follows:

1. A small light source at L is observed in air initially through a small opening at E with the eye placed at O.
2. The medium is placed in position with E on or very close to the smooth horizontal surface, and again the image of the light from L is observed - now refracted at the surface, and passing through E along EN. The observer considers the image to lie at M, which can be found using the parallax method, [by simultaneously viewing a small object placed outside the medium, and adjusting to give the same height ML, when there is no relative motion on moving the eye slightly]. Gregory regards the angle of deviation NEO as a measure of the refraction: the same experiment survives to this day, where one measures the true depth and the apparent depth of an object, from which the refractive index of the medium can be extracted.

§2. Prop.2.3.

Prop. 2. Problema.

Refractiones cujuscunq; diaphani aere invenire.

Sit planum aliquod ABCD stabilitum, & horizonti parallelum, vel eo circiter; sitq; punctum aliquod firmissimè stabilitum in sublimi positum E, & per punctum E concipiatur duci perpendicularis ad horizontem VEG, sitque angulus incidentiae in diaphano (cujus anguli refractionis requiritur) GEL, qui facit è mensuratur astrolabio, vel quadrante: & in puncto L, figatur corpusculum resplendens, & tandem impleatur totum illud spatium inter planum ABCD, & punctum E, diaphano optimè polito, cujus superficies ad E punctum, horizonti sit exquisitè parallela, &

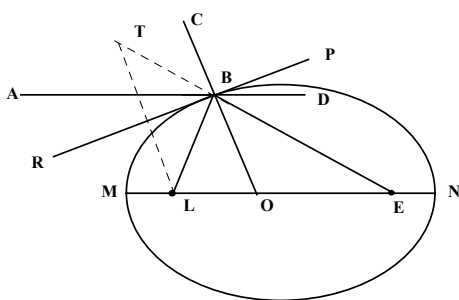
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aspicienti per punctum E, apparebit resplendens in M; explorato igitur angulo NEV, seu GEM; differentia inter illum, & priorem angulum OEV, seu GEL dabit angulum refractionis quaesitum NEO, competentem

§2. Prop.3.1.

Prop. 3. Problem.

Let the two angles be given, ABC the larger and DBE the smaller, and the tangent line



RBP of the ellipse shall be found perpendicular to CB at the point B, the line AB shall be parallel to the axis of this ellipse, and the line BE shall cross through the further focus. Through any point of the line EB [*i.e.* the actual size of the ellipse is not important], without doubt E, EO is drawn parallel to the line AB, to which the other line CB is produced in O, and the angle OBL is made equal to the angle OBE, and MN shall be equal to the sum of EB and

Conversely, since LB and BE together are equal to the axis MN (by the converse of [Prop.] 48, *Book 3, Apollonius*), and since the angles OBE and OBL are equal, if they are taken from the right angles RBO and OBP,

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then the equal angles EBP and RBL are left; and therefore the line RBP is made to touch the ellipse in the point B (by the converse of [*Prop.*] 52, *Book 3, Apollonius*). With AB parallel to the axis MN, the angle RBA is the complement of the given larger angle ABC; and because the lines AD and MN are parallel, the angle to the further focus BEO is equal to the given smaller angle DBE, as required.

§2. Prop.3.2.

Note on Prop. 3.

Angle ABC is the angle of incidence i , while Gregory has taken the angle of deviation DBE or d , as a measure of the refraction by the medium. Thus, the experimental procedure of Prop. 2 for measuring refraction is adopted for the curved surfaces of the lenses to be subsequently discussed.

§2. Prop.3.3.

Prop.3. Problema.

Datis duobus angulis, non obtusis, invenire ellipsin, ut linea axi parallela, in eam incidens, efficiat cum tangente angulum, equalem complemento majoris, & recta a puncta incidentiae ad focum maxime distantem, efficiat cum axe angulum aequalem minori.

Sint dati duo anguli, ABC major, DBE minor, sitque invenienda ellipsis tangens lineam RBP ad CB perpendicularem in puncto B, cujus axis rectae AB sit parallelus, & linea BE per focum maxime distantem transeat. Per punctum quodlibet lineae EB, nimirum E, ducatur lineae AB parallela, EO, quae utrinque producat: producat CB in O, & fit angulus OBL aequalis angulo OBE, fitque MN aequalis EB & BL simul; quam dico esse axem ellipseos quaesitae, positis focus L & E: axe MN, & focus L, E, describatur ellipsis MBN quae necessario transibit per punctum B; quoniam LB, BE simul sint aequales axi MN (per conversum [Prop.] 48, lib 3, Apoll.), & quoniam anguli OBE, OBL sunt aequales, si a rectis RBO, OBP auferantur,

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relinquuntur anguli EBP, RBL aequales; tangit igitur linea RBP ellipsin in puncto B (per conversum [Prop.] 52, lib 3, Apoll.); facitq; cum AB, axi MN parallela, angula RBA aequalem complemento anguli dati majoris ABC; & ob parallelisimum linearum AD, MN, angulus ad focum remotiorem BEO, aequalis est angulo dato minori DBE, quod erat faciendum.

§2. Prop.4.1.

Prop. 4. Theorem.

With the same situation, I say that the sine of the difference of the given angles shall be to the sine of the larger angle, as the separation the foci to the ellipse axis.

For the line EB [see Prop.3 -Fig.1] may be produced to T, and BT made equal to BL, and TL drawn. Therefore the angles BTL and BLT are equal, and also the angle LBE is equal to the sum of both, and therefore EBO, or half the angle EBL, is equal to the angle BTL, therefore the triangles EBO are ETL are similar. But the angle BOL is equal to the larger given [i] ABC, on account of the parallel lines AB and MO, and the angle BEO is equal to the smaller given angle DBE [d]. But BEO and OBE added together are equal to the angle BOL, and therefore the angle OBE, or LTE is equal to that angle, and this is the difference of the given angles ABC and DBE; and also the angle TLM is equal to the angle BOL, or to the given larger angle ABC.

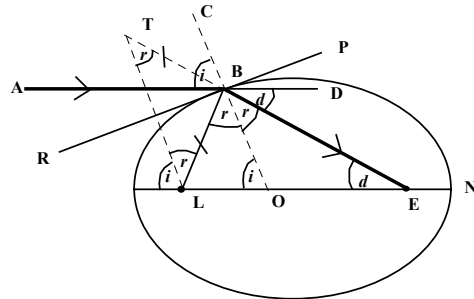
[11]

As a consequence we conclude the sine of the difference of the angles given, that is the sine of the angle LTE is to the sine of the larger angle given TLM, as the separation of the foci LE is to the length of the axis of the ellipse TE. Q.E.D.

§2. Prop.4.2.

Note on Prop. 4.

Gregory has independently discovered a form of the familiar law of refraction $\sin i / \sin r = n$, where i is the angle of incidence ABC, r the angle of refraction OBE, and n the index of refraction of the medium relative to air. For the angle of deviation d used by Gregory is given by $d = i - r$, while n is related



Prop.4 -Fig.1.

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to the eccentricity e of the ellipse of major diameter $2a$ by $n = 1/e$. Thus, $\sin i/\sin(i - d) = 2a/2ae$, or $\sin i/\sin r = 1/e$, where e is the eccentricity of the ellipse in modern terms, though this particular terminology was not in use at the time. Indeed, the focus/directrix property of conic sections was not discovered until the beginning of the 19th century by the two Belgian mathematicians Quetelet and Dandelin - see e.g. Eves: *An Introduction to the History of Mathematics*, p. 169.

§2. Prop.4.3.

Prop. 4. Theorema.

Isidem positis, dico sinum differentiae angulorum datorum, esse ad sinum anguli majoris, ut focorum distantia, ad axem ellipseos.

Producatur enim linea EB in T; fitque BT aequalis BL, & ducatur TL; erunt igitur anguli BTL, BLT aequales, & LBE aequales ambobus simul, ergo & EBO semissis anguli EBL, aequalis erit angulo BTL, triangula igitur EBO, ETL sunt equiangula; est autem angulus BOL aequalis majori dato ABC, ob parallelismum linearum AB, MO, estque angulus BEO aequalis minori angulo dato DBE: BEO autem & OBE sunt aequales angulo BOL; igitur angulus OBE, vel illi aequalis LTE, est differentia angulorum datorum ABC, DBE; est quoque angulus TLM aequalis angulo BOL, vel majori dato ABC. Concludimus

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ergo, sinum differentiae angulorum datorum, hoc est anguli LTE, esse ad sinum anguli majoris dati nimirum TLM, ut distantia focorum LE, ad axem ellipseos TE, quod erat demonstrandum.

§2. Prop.5.1.

Prop. 5. Theorem.

For the same situation, if the larger angle were the angle of incidence of some ray from the rare to the dense transparent medium, and the smaller angle agreeing by refraction with the said angle of incidence, and the ellipse found that forms the surface of refraction from the same rare medium into the dense. I say that all the rays parallel to the axis of the ellipse, and incident on the ellipse, are refracted at the points of incidence, and are concurrent at the focus: moreover this ellipse may be called the ellipse of the aforementioned dense medium.

From the above analogous discussion, it is seen the parallel rays in one plane of the rare medium meet in one point of the dense medium, only if the surface of refraction shall be a certain ellipse with fixed measurements, which is appropriate for the density of the medium. Since indeed the circle gathers together the parallel rays from the rare medium into a single point of the densest medium; the parabola certainly gathers together the parallel rays from one medium into an imaginary point of another medium of the same density, standing apart at an infinite separation; therefore it follows for the ellipse, which is intermediate between these figures, that the parallel rays in the plane of the rare medium are gathered together in a single point of the medium of intermediate density.

Therefore with these things touched on, we may undertake the experimental demonstration of this Theorem; and we may suppose that it is truth, in order that it may be revealed to be absurd (if an absurdity should lie hidden within it). Witelo observed the refraction of water [*i.e.* the angle of deviation], agreeing with an angle of incidence in air 30° : to be $7^\circ 30'$. From this observation by the proceeding theory, we find the dimensions of the ellipse thus shall be as 50000, the sine of the angle of incidence 30° , to 38268, the sine of the angle of refraction $22^\circ 30'$: [or more conveniently] 10000 the axis of the ellipse, to 7654 the separation of the foci. Hence by the same Theorem, we can compute

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the rest of the angles of refraction: as 10000 - the axis of the ellipse is to 7654 - the separation of the foci, as 17365 the sine of the angle of incidence 10^0 , is to 13291 the sine of the angle of refraction: $7^0:38'$, which taken from the angle of incidence 10^0 leaves the angle of refraction

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$2^0:22'$ equal to the difference from the observed angle, $2^0:15'$, except for $7'$. In this way we have computed the rest that follow. But these disagreements [in values] need not disturb the reader, for these reasons. Indeed from the first, from the observations of Witelo, it appears that it was enough to have observed only to [the nearest] half degree. And (as Kepler observes in *Astron. Opt. fol.* p.116.), he is sure from his own trials on mastering refraction, by putting his hand to it, he might reduce these to order through the equality of the second increments [Gregory, however, does not involve himself in Kepler's fitting scheme]: All of these observations indeed increase with increments of $30'$ [by this statement, Gregory appears to mean Witelo's angles are measured to within a half or even a quarter of a degree: Gregory's own data in Table 5.3, shows accuracy to the nearest minute].

The refraction observations from Witelo; and our calculated discrepancies of these.

Angles. of incidence in air.	Refraction by water from air.		Diff.	The refraction of glass from air.			The refraction of glass from water.				Angles of incidence in water
	Obs.	Calcul.		Obs.	Calcul.	Diff.	Obs.	Calcul.	Diff.		
0	0	0		0			0				
10	2 15	2 22	+ 7	03 00	03 20	+ 20	00 30	01 12	+ 42	10	
20	4 30	4 50	+ 20	06 30	06 48	+ 18	01 30	02 27	+ 57	20	
30	7 30	7 30	0 0	10 30	10 30	00 00	03 00	03 50	+ 50	30	
40	11 0	10 32	- 28	15 00	14 35	- 25	05 00	05 28	+ 28	40	
50	15 0	14 6	- 54	20 00	19 14	- 46	07 30	07 30	00 00	50	
60	19 30	18 29	- 61	25 30	24 40	- 50	10 30	10 12	- 18	60	
70	24 0	24 1	- 29	31 30	31 08	- 22	14 00	14 02	+ 02	70	
80	30 0	31 5	+ 65	38 00	38 58	+ 58	18 00	19 34	94	80	

[Prop.5 -Table 1]

The observed refractions from Arthansius Kercher, & our method following calculated with differences.

Angles of incidence in air	Refraction of water			Refraction of wine			Refraction of oil			Refraction of glass		
	Obs.	Calc.	Diff.	Obs.	Calc.	Diff.	Obs.	Calc.	Diff.	Obs.	Calc.	Diff.
	0 °	0 °	,	0 °	0 °	,	0 °	0 °	,	0 °	0 °	,
	2 20	2 12	+ 8	2 30	2 28	- 2	2 50	2 33	- 17	3 10	3 27	+ 17
	4 38	4 56	+ 18	4 45	5 3	+ 10	5 10	5 12	+ 2	6 40	7 1	+ 21
	7 40	7 40	0 0	7 50	7 50	0 0	8 4	8 4	0 0	10 50	10 50	0 0
	11 9	10 45	- 24	11 4	10 59	- 5	11 50	11 18	- 32	15 8	15 2	- 6
	15 6	14 24	- 42	15 10	14 41	- 29	16 10	15 5	- 65	20 12	19 48	- 24
	19 40	18 50	- 50	19 50	19 12	-38	20 20	19 42	- 38	25 50	25 20	- 30
	24 49	24 26	- 23	24 50	24 50	0 0	25 12	25 25	+ 13	31 10	31 54	+ 44
30 4	31 33	+ 89	30 10	32 0	+110	30 54	32 38	+104	38 10	39 43	+ 93	

[13]

But in the work of Kercher from the differences of the observations [Prop.5 -Table 2], there is not even the shade of order: thus it shall be beyond all doubt, that these observations are in error. And not without wonder, it will seem (if anyone should consider the thing more accurately) that certain differences had crept in, with so subtle an enquiry; where a trivial error in the base of the calculations is multiplied in those

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Refraction of spring water from our own Observations with the Calculation of their Differences.			
Ang. of incidence in air.	Refract. observed.	Refraction calculated.	Differences
0	0	0	
13 28	03 28	03 25	- 03
26 48	05 48	07 03	+ 15
41 50	11 50	11 50	00 00
59 12	19 12	19 07	- 05
71 20	26 20	26 05	- 15

[Prop.5 -Table 3]

proceeding. But the truth of this Theorem has been apparent many times from our different trials: just as it will be evident from this single example, which from our angles of refraction we have measured for spring water in our youth; where reliably by our first problem, and finely enough (from the size of the instrument), for the angles of incidence in water 10^0 , 20^0 , 30^0 , 40^0 , 45^0 . [These angles do not correspond with those in Prop.5 -Table 3.]

If truly these observations made with great care do not give satisfaction: Come mathematicians! And from more subtle observations confirm this most beautiful speculation of refraction.

Corollary

It follows from this theorem, the angle between the line of refraction with the axis of the dense ellipse, (see the angle BEO in the figure above), shall always be the angle of refraction, meeting the angle of incidence ABC. But now we can explain the rest geometrically from the aforementioned analogy with the help of theorems.

§2. Prop.5.2.

Prop. 5. Theorema.

Iisdem positis; si angulus major fuerit angulus incidentiae alicujus Radii diaphano raro in densum, & angulus minor refractionis dicto angulo incidentiae competens, fueritq, ellipsis inventa, superficies refractionis ex eodem diaphano raro in densum; dico omnes radios axi ellipseos parallelos, & in ellipsis incidentes, in punctis incidentiae Refringi, & in focum concurrere: Vocetur autem haec ellipsis, ellipsis densitatis praedictorum diaphanorum.

Ex superiore discursu Analogico, videtur radios parallelos in uno plano diaphani rari, congregari in unum punctum diaphani densioris, si modo, superficies refractionis fit ellipsis certa cujusdam dimensionis, quae diaphanorum densitati conveniat. Quoniam enim circulus congregat Radios Parallelos e medio raro in unum punctum medii densissimi; Parabola vero congregat radios parallelos, ex uno diaphano in punctum imaginarium, alterius diaphani ejusdem densitatis, infinite distans; Sequitur igitur ellipsim, quae media est inter hasce figuras, radios parallelos in plano diaphani rari congregare in unum punctum diaphani mediocriter densioris. His igitur praelibatis, ad demonstrationem hujus Theorematis experimentalem accedamus; & supponamus verum esse, ut absurdum (si quod lateat) patefiat. Observavit Vitellio refractionem aquae, competentem angulo incidentiae in aere 30^0 : esse $7^0 30'$, & hac observatione per praecedens Theorem, ita inveniemus ellipseos dimensiones, fit ut 50000: sinus anguli incidentiae 30^0 : ad 38268 sinum anguli refracti $22^0 30'$: 10000 axis ellipseos, ad 7654 distantia focorum. Et eodem Theoremate reliquas refractiones ita Computemus; ut 10000 axis ellipseos ad 7654 the distantiam focorum, ita 17365 sinum anguli incidentiae 10^0 , ad 13291 sinum anguli refracti: $7^0 38'$: qui ablatus ab angulo incidentiae 10^0 : relinquet angulum refractionis [12] $2^0 22'$: differentem ab observatione, $2^0 15'$, nisi $7'$: atque ita sequentia Computavimus. Sed lectorem ne moveat haec discrepantia, ob has rationes: Primo enim, satis apparet observationibus Vitellionis eum observasse tantummodo ad graduum semisses; Et (ut notat Keplerus in *Astron. Opt. fol.* 116.), certum est suis ab experientia captis refractionibus, manum admovisse, ut in ordinem illas, per secundorum incrementorum aequalitem, redigeret: [13] Omnia enim harum observationum incrementa surgunt per differentiis $30'$. In Kercheri autem observationum differentiis nulla est vel ordinis umbra; ita ut extra omne dubium sit, illius observationes esse fallaces. Nec sane mirum, videbitur (si quis rem accuratius intueatur) differentias quasdam in tam subtili disquisitione irrepsisse; ubi levis error in calculationum radice, in processu multiplicatur. Huius autem Theorematis veritas, per varia experimenta nobis multoties emicuit: veluti ex hoc unico, quod ad aquam fontanam habuimus, apparebit; ubi per primum nostrum problema fideliter, & (pro instrumenti magnitudine) satis subtiliter, ad angulos incidentiae in aqua 10^0 , 20^0 , 30^0 , 40^0 , 45^0 : tales juvenibus refractiones. Si vero curiosi ingenii hae

observationes non satisfecerint; agite Mathematici, & subtilioribus observationibus, hanc pulcherrimam Dioptrices speculationem confirmate.

Corollarium

Sequitur ex hoc theoremate, angulum lineam refractionis cum axe ellipseos densitatis, (v.g. in superiore figura angulum BEO), semper esse angulum refractionis, Competentem angulo incidentiae ABC. Restat autem nunc ut quod superest, ex praedicta analogia, ope theorematis, geometrice demonstremus.

[14]

§2. Prop.6.1.

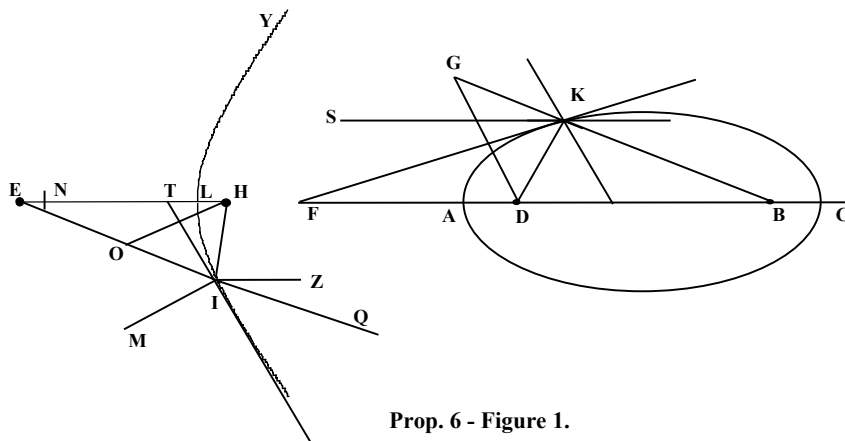
Prop. 6. Theorem.

If the ratio of the length of the axis to the inter-focal distance of a dense ellipse between two media is thus as the ratio of the inter-focal distance to the length of its axis of a hyperbola, and if the hyperbola is the surface of refraction of the rays arising from its own outer focus in the rarer medium, then all the given rays are refracted parallel by the surface of the denser medium. This hyperbola is called the dense hyperbola of the given media.

The ratio is AC to BD, the axis length of the dense ellipse between any two media AKC to the separation of the foci, and this is thus as EH to LN, the separation of the foci of the hyperbola to the axis length of the same. The hyperbola ILY is the surface of refraction. The ray EI is incident on a branch of the hyperbola coming from the far focus, and passing from the rarer to the denser medium at

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the point I. The ray EI is refracted through Z, thus QIZ is the angle of refraction [i.e. the deviation d]. I say that IZ is parallel to the axis of the hyperbola ENLH. For the line IT is



drawn through the point I, tangent to the hyperbola ILY at the point I. A perpendicular line IM is drawn from the point I, and EIM is the angle of incidence, which is equal to the angle GDF from the focus D of the ellipse [i.e. equal to the angle of incidence for the ellipse: see the note following], and the line BG is drawn equal in length to the axis AC; from the preceding corollary it is apparent that the angle GBD is equal to the angle of refraction QIZ [i.e. the angle of deviation d], meeting the angle of incidence GDF, or EIM: and thus IO is made equal to the line IH, and OH is joined; and EO is equal to the length of the axis of the hyperbola, and IT dividing the angle HIO in two equal parts is

perpendicular to OH (*Apol.* 3.51 & 3.48). But IT is perpendicular to IM, and therefore IM and OH are parallel, and hence the angle EIM is equal to the angle IOH: but EIM is equal to the angle GDF, and therefore IOH is equal to the same GDF. Therefore in the [similar] triangles EOH, BDG the two sides HE, EO are proportional to the two sides GB and BD, [thus, the inter-focal distance for the ellipse is proportional to the vertex separation of the hyperbola; while the vertex separation of the ellipse is proportional to the inter-focal distance for the hyperbola: conjugate conics], and the angle GBD is equal to the angle HEO, *i.e.* the angle QIZ is equal to the angle QEH, IZ and EH are therefore parallel. Q.e.d.

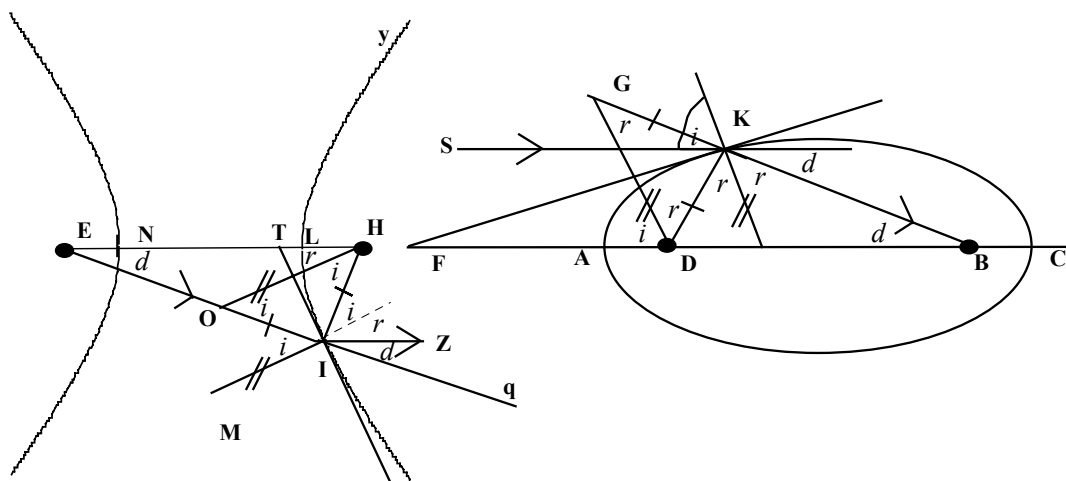
Scholium.

It is also possible to find the hyperbola by trial first, and then the ellipse is deduced from this by a geometrical demonstration. But we have composed these theorems in the same order in which they were found by us.

And from these surfaces the angles of refraction are measured in the clearest analogous manner which you can show - through tests with which you can steadfastly agree, and by geometrical demonstrations in three dimensions that you can prove. It now remains that we may explain optical devices, before [demonstrated by others] through approximation, now shown with geometric precision by us. Since truly, up to this point we have said so much about the surfaces of refraction, it is necessary that we should present some other Lemmas, with the help of which (without doubt of such kinds that constitute all the machinery of optics) one is allowed to proceed from surfaces to solids.

§2. Prop.6.2.

Notes on Prop. 6.



Prop. 6 - Figure 2.

From Prop. 6 - Fig.2, it is seen that the triangle associated with the ellipse, BDG, is similar to the corresponding triangle EOH for the hyperbola. Hence if e , where $0 < e < 1$, is the eccentricity of the ellipse with major diameter $2a$, then $AC = BK + KD = BG = 2a$,

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and $BD = 2ae$; similarly, for the hyperbola with eccentricity $e' > 1$, $NL = EI - IH = EO = 2a'$, and $EH = 2a'e'$.

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§2. Prop.6.3.

Prop. 6. Theoremata.

Si fuerit; ut axis ellipseos densitatis duorum diaphanorum, ad distantiam focorum ejusdem, ita Distantia focorum Hyperbolae, ad sui axem; fueritq: hyperbola superficies refractionis Radiorum ex foco suo exteriore, in diaphano rariore existente : Omnes dicti radii, per refractionem in superficie densioris diaphani, ad Parallelismum reducentur. Vocetur autem has hyperbola, hyperbola Densitatis praedictorum Diaphanorum.

Sit ut AC axis Ellipseos densitatis duorum diaphanorum quorumcunq ; AKC; ad BD, distantiam focorum; ita EH, distantiam focorum hyperbolae, ad LN, axem ejusdem: sitq hyperbola ILY superficies refractionis, in quam incidat Radius EI ex foco exteriore E, in diaphano rariore existente, in punctum I, existens in superficie diaphani densioris, & refrangatur linea EI in Z, ita ait angulus QIZ, sit angulus refractionis : Dico IZ esse parallelam axi hyperbolae ENLH. Ducatur enim per

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punctum I, linea IT, tangens Hyperbolam ILY in puncto I, & a puncto I ducatur linea IT perpendicularis, IM, eritq; EIM angulus incidentiae, cui fiat equalis ex foco ellipseos D, angulus GDF, & ducta recta BG, axi AC aequali; ex praecedente corollario evidens est, angulum GBD esse aequalem angulo refractionis QIZ, competentem angulo incidentiae GDF, seu EIM: fiat itaq; IO aequalis rectae IH, & jungatur OH; eritque EO aequalis axi, hyperbolae NL, & IT dividens angulum HIO bifarium (*Ap.3.51, Ap.3.48*), est perpendicularis ad OH; sed & perpendicularis est ad IM; igitur IM, OH sunt parallelae, & angulus EIM aequalis angulo IOH: sed EIM, est aequalis angulo GDF, igitur & IOH eidem GDF est aequalis. In triangulis igitur EOH, BDG duo latera HE, EO sunt proportionalia duobus lateribus GB, BD, & angulus HOE, aequalis angulo GDB, ergo & angulus GBD, est aequalis angulo HEO, hoc est angulus QIZ, aequalis angulo QEH, parallelae igitur sunt IZ & EH, quod erat demonstrandum.

Scholium.

Poterat etiam & hyperbola densitatis per experientiam primo inveniri, & ellipsis demonstratione Geometrica ex ea deduci; Nos autem, haec scripsimus eadem Methodo, qua a nobis reperta sunt.

Atque hisce de superficiebus, quae Refractiones metiuntur per analogiam clarissime Monstratis, per experientias firmiter probatis, & per demonstrationem Geometricam solide confirmatis : Restat nunc ut Machinas Opticas, ante per approximationem, nunc Geometrice demonstremus. Quoniam vero, hactenus de superficiebus refractionum tantum, loquuti sumus; oportet ut aliquot Lemmata praemittamus, quorum ope a superficiebus ad solida (qualia nimirum sunt omnia Machinamenta Optica) liceat Progredi.